

# Catenary and autotransformer coupled optimization for 2x25kV systems planning

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## Abstract

In the design process of a 2x25kV power supply, the location and sizing of autotransformers and the choice of the catenary type are usually two aspects very closely interrelated. As a result, the number of autotransformers can normally be reduced or augmented if the catenary is upgraded or downgraded respectively. In this paper, coupling equations between these two aspects are described in detail.

These equations are used to formulate a multi-attribute optimization problem in which the global efficiency of the investment is optimized. In this problem, the considered attributes are: (i) investment cost and (ii) a performance index that is defined in the paper. The optimization procedure gives a reduced set of catenary and autotransformers combinations that maximize efficiency. This optimal reduced set can be used as an input in the design process of power supply. For the solution of this problem, different dominancy criteria are evaluated. Furthermore, sensitivity studies are carried out during the optimization process to help in the search of catenary and autotransformers combinations.

As a study case, the proposed optimization procedure has been used to obtain a reduced set of catenaries and autotransformers based in the catenary C-350, which has been used in the Madrid – Barcelona – French border new high-speed line.

*Keywords: power supply system, high speed railways, multi-attribute optimization, long-term infrastructure planning.*

## 1 Introduction

In the design process of a 2x25kV power supply, the location and sizing of autotransformers and the choice of the catenary type are usually two aspects very



closely interrelated. As a result, the number of autotransformers can normally be reduced or augmented if the catenary is upgraded or downgraded respectively. In this paper these relationships are analyzed and used to formulate a multi-attribute optimization problem in which the global efficiency of the investment is optimized [1].

Section 2 describes the power supply system of AC-electrified railways. In section 3 an equivalent model is used to represent bi-voltage  $2 \times 25 \text{ kV}$  and to analyze the relationships between catenary parameters and autotransformers separation. In section 4 the optimization problem is described. In section 5 the optimization procedure is applied to a real case. Finally, in section 6 the conclusions of this work are presented.

## 2 The power supply of AC electrified railways

### 2.1 General structure

Figure 1 shows the general structure of power-supply systems of AC electrified railways:

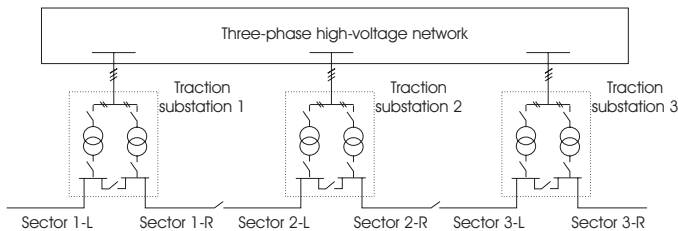


Figure 1: Structure of the power supply system.

As shown, the electrical system is divided in electrically-isolated single-phased sectors, which are fed from the three-phase network through a traction substation. These substations are connected between two of the three phases of the high-voltage network. Each of these sectors can use either mono-voltage system ( $1 \times 25 \text{ kV}$ ) or bi-voltage system ( $2 \times 25 \text{ kV}$ ). In mono-voltage systems [2], the feeding conductors are set to the specified voltage level (see Figure 2).

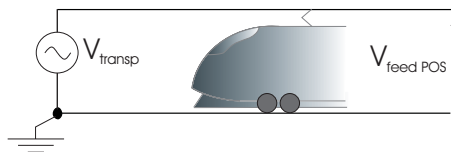


Figure 2: Mono-voltage system configuration.

In bi-voltage systems, a higher voltage is set between feeding conductors [3, 4]. This voltage is reduced by using autotransformers distributed along the

catenary (see Figure 3). In these systems, the term cell very often refers to the portion of catenary located between two consecutive autotransformers. Typical values for cell lengths are 10-15km.

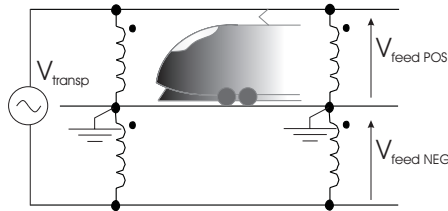


Figure 3: Bi-voltage system configuration.

As this paper is focused on this system, it is assumed that all the sectors are fed using bi-voltage system.

## 2.2 Catenary

The typical configuration of the catenary of an AC railway line is shown in Figure 4. The catenary contains several physical conductors that can be grouped into three groups: positive, negative and ground wires. In case of multiple tracks, other conductor arrangements are possible.

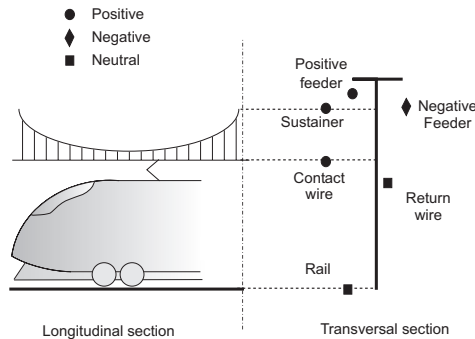


Figure 4: Typical conductor distribution.

The positive wires are the positive feeder, the sustainer wire and the contact wire. There is usually only one negative wire called negative feeder. The ground wires are the rail, the collector wire and the return wire.

The conductors of each group are connected between them at regular intervals (typically 300m). Additionally, ground conductors are frequently connected to earth.

### 3 The effect of autotransformers and catenary upgrades

#### 3.1 Base magnitudes

In order to improve numerical stability, normally per unit magnitudes are used to carry out all the necessary calculations. Thus, the circuit can be divided into three zones based on their nominal voltage. Figure 5 shows the considered zones: (i) high-voltage zone, (ii) positive zone and (iii) negative zone.

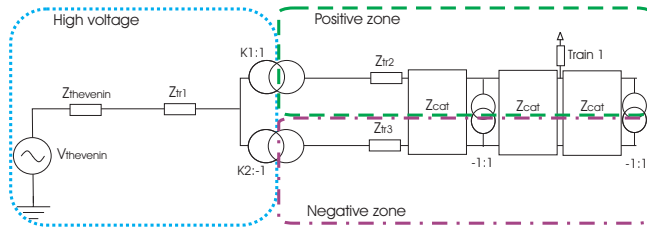


Figure 5: Zone division for base magnitudes selection.

A base power  $S_{base}$  has to be chosen and is common to all the zones (a typical value is 10MW). Furthermore, base voltages have to be selected for the three zones. If base voltages are exactly the voltages of every zone in a scenario without any kind of load, transformation ratios take values of 1 and  $-1$ . Base impedance and base currents can be determined from the base power and voltage of each zone.

#### 3.2 The 1x25kV equivalent model of 2x25kV systems

In [5] the behavior of bi-voltage systems is analyzed and a equivalent model is proposed to represent bi-voltage 2x25kV systems as if they were mono-voltage 1x25kV. This is the model used in the presented work.

Figure 6 shows the approximated behavior of the circuit with a train consuming a current  $I$ , assuming: (i) that voltage drop along a cell in the positive and in the negative side have the same value but different sign and (ii) that, as far as autotransformers can be supposed ideal, it can be assumed that there are current flows only in the autotransformers that are immediately adjacent to the considered train.

In this figure  $V_{cell\ n}$  is the voltage drop along the cell  $n$ ,  $I_{p,trans}$  and  $I_{n,trans}$  are respectively the positive and negative currents in the transmission cells,  $I_{p,train}$  and  $I_{n,train}$  are respectively the positive and negative currents in the cell of the train,  $L_{cell}$  is the length of the cell of the train,  $x$  is the relative position of the train, expressed as a fraction of  $L_{cell}$ .

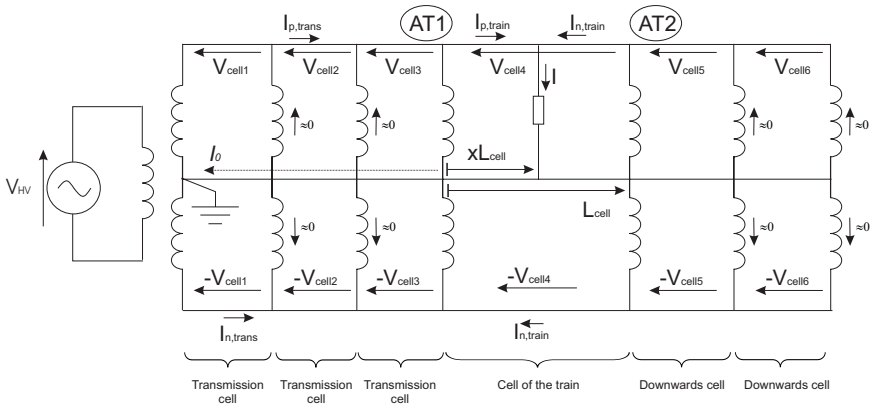


Figure 6: Approximated behavior of bi-voltage system.

Based on these simplifications, the positive phase of the bi-voltage system can be represented as shown in Figure 7. In this model, two different contributions have been identified: (i) the equivalent impedance of the catenary  $z_{eqv,cat}$  that depends only on the configuration of physical conductors and (ii)  $z_{gap}$  that is associated to the separation between autotransformers.

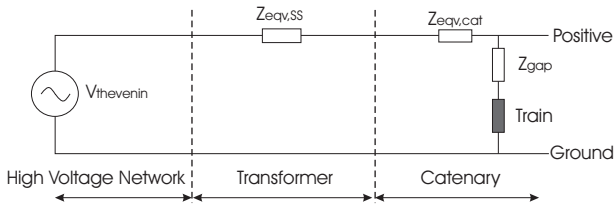


Figure 7: Mono-voltage equivalent circuit of bi-voltage system.

The parameters of this equivalent circuit are calculated as follows:

$$z_{eqv,cat} = \tilde{z}_{eqv,cat} \cdot D_{ss,train} \tag{1}$$

where the symbol  $\sim$  is used to refer per length unit magnitudes and  $D_{ss,train}$  is the distance between the substation and the train

The equivalent impedance  $\tilde{z}_{eqv,cat}$  of the catenary can be obtained from the elements of the equivalent conductors impedance matrix [6], where the sub-indexes  $p$  and  $n$  represents the positive and negative conductors respectively.

$$\tilde{z}_{eqv,cat} = \frac{\tilde{z}_{pp} \cdot \tilde{z}_{nn} - \tilde{z}_{pn} \cdot \tilde{z}_{np}}{\tilde{z}_{pp} + \tilde{z}_{nn} + \tilde{z}_{pn} + \tilde{z}_{np}} \tag{2}$$

The impedance gap  $z_{gap}$  associated to the separation of autotransformers can be obtained as follows:

$$z_{gap} = L_{cell} \cdot x(1-x) \frac{\tilde{z}_{pp}^2 + \tilde{z}_{pp}\tilde{z}_{np} + \tilde{z}_{pn}\tilde{z}_{pp} + \tilde{z}_{pn}\tilde{z}_{np}}{\tilde{z}_{pp} + \tilde{z}_{pn} + \tilde{z}_{np} + \tilde{z}_{nn}} \quad (3)$$

The voltage drop associated to the impedance  $z_{gap}$  is referred as voltage deviation from the equivalent model of the catenary. As shown in Figure 8, this voltage deviation starts and ends in cell in which is located the train. In other words, at the end of this cell all the voltage deviation is recovered and thus no extra voltage drop has to be added to the trains that are located downwards.

Figure 8 summarizes the voltage drops in the sector in a scenario with only one train:

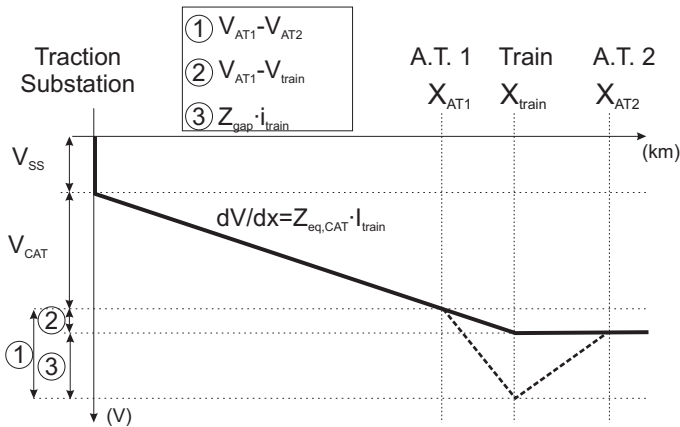


Figure 8: Voltage drops in a bi-voltage sector.

It can be seen that the deviation impedance is proportional to the distance between autotransformers. Consequently, as far as the number of autotransformers is increased, the relative weight of the deviation is reduced.

### 3.3 Influence of catenary type and autotransformers distance on voltages

Using the described model, sensitivity of the voltage drops to catenary upgrades or to autotransformer additions can be analyzed. These are two common investment decisions to be taken in the design process of the power supply in bi-voltage 2x25kV electrified railways.

Figure 9 shows the effect of upgrading the catenary in the voltage drops along the catenary  $\Delta V_{cat}$ . When upgrading the catenary, the most important effect is a reduction of voltage drops all along the sector. Additionally, the voltage drop due the separation autotransformers (corresponding to  $z_{gap}$ ) can also be reduced.

Figure 10 shows the effect of shortening the distance between consecutive autotransformers, which is typically achieved by adding extra autotransformers to the sector. Unlike the catenary upgrade, the benefits of this enhancement are limited to the cell whose distance has been reduced, due to the local influence of voltage deviations  $V_{DESV}$ .

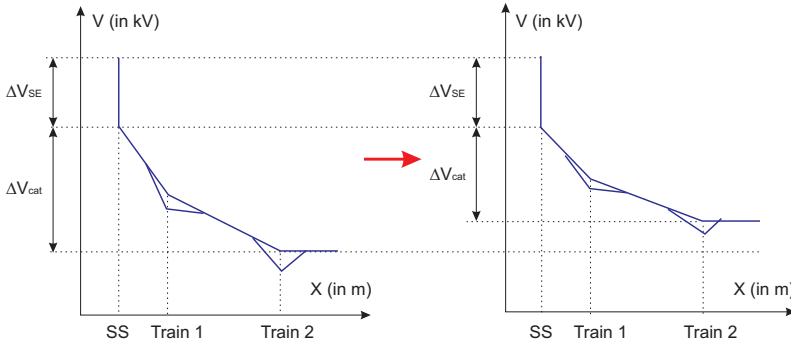


Figure 9: Effect of upgrading the catenary.

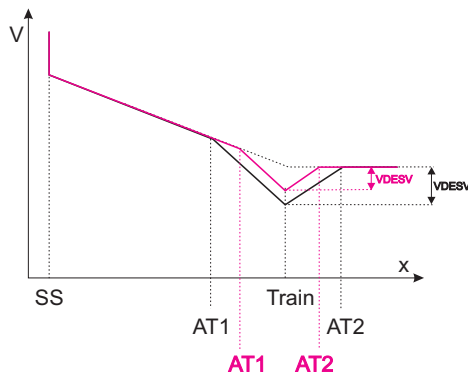


Figure 10: Effect of shortening length of cells.

As it has been described, both catenary upgrades and autotransformer additions can be used to reduce voltage drops.

#### 4 The optimization problem

In the design process of AC-electrified railways, voltage limitations are commonly active, especially when evaluated in degraded situations (substation out of order). Thus, determining the most efficient way of reducing voltage drops is a key factor in the design of the power supply.

As it has been described, catenary upgrades and autotransformer additions are often investment decisions that are exchangeable in order to reduce voltage drops, from a technical point of view. Therefore, economical criteria have to be considered to determine the most efficient combination of catenaries and autotransformer distributions. For that reason, a multi-criteria optimization problem has been formulated and solved.

The goal of this optimization is to obtain a reduced repository of combinations of catenary types and autotransformer distributions, in which the efficiency of its elements is maximized. This set is to be used as an input in the

design of the power supply system. To determine this repository, the considered attributes are: (i) investment cost and (ii) a efficiency index that is to be defined. In order to evaluate the efficiency of each {catenary, autotransformers} combination the efficiency index **ZEQ** is defined in eqn. (4). This index corresponds to the total equivalent impedance seen between the substation output and a train located in the further cell of the corresponding sector. In order to simplify the resulting expression, it has been assumed that autotransformers are uniformly distributed.

$$\mathbf{ZEQ} = \tilde{\mathbf{z}}_{eq,cat} \left[ \frac{n-1}{n} L + \frac{L}{n} x \right] + \mathbf{z}_{gap} \quad (4)$$

where  $n$  is the number of autotransformers of this sector and  $L$  is the length of this sector. Expanding  $\mathbf{z}_{gap}$  in eqn. (4) becomes:

$$\mathbf{ZEQ} = \tilde{\mathbf{z}}_{eq,cat} \left[ \frac{n-1}{n} L + \frac{L}{n} x \right] + (\tilde{\mathbf{z}}_{pp} - \tilde{\mathbf{z}}_{eq,cat}) x (1-x) \frac{L}{n} \quad (5)$$

As **ZEQ** depends on the relative position  $x$  of the train in its cell, one of the following criteria for can be assumed:

- The train is located in the middle of its cell ( $x = 0.5$ )
- The train is located where the index **ZEQ** reaches its maximum.

It should be noted that **ZEQ** incorporates the effect of both catenary type and autotransformers in an equilibrated manner, as it corresponds to their contribution in the total voltage drops.

The considered cost function is:

$$C_{cat,n} = C_{cat} \cdot L + C_{AT} \cdot n \quad (6)$$

where  $C_{cat,n}$  is the cost associated to catenary *cat* with  $n$  autotransformers,  $C_{cat}$  is the per length unit cost of the catenary *cat* and  $C_{AT}$  is the unitary cost of each autotransformer.

To get the optimal repository the following steps have to be accomplished:

- Exploration of the possible catenary/autotransformer configurations. As the sustainers and the contact wires are fixed, the number of combinations is not excessive. Symmetry restrictions can also be used to reduce the search space.
- Eliminate dominated configurations (a configuration is dominated if it is worse in cost and in efficiency than another configuration). Relaxed dominancy criteria can also be applied in order the eliminate configurations (i) a bit cheaper but much lower efficiency or (ii) a bit more efficient but much more expensive.
- Within the resulting repository, chose the final configurations trying to cover uniformly the costs range.



## 5 Study case

The proposed optimization procedure has been used to get a repository of catenaries and autotransformers based on the catenary C-350 that has been used in the Madrid – Barcelona – French border new high-speed line. The considered costs are summarized in Table 1.

Table 1: Costs structure of the study case.

Description	Cost	Observations
Autotransformer	200 units	Cost per autotransformer
Fixed cost of catenary	29 units/km	Includes installation of catenary towers, insulators, contact wires and sustainers
Cost per added feeder	0.3 units/km/feeder	Only in configurations with positive or negative feeder
LA-110 conductor	0.716 units/km	To be included only if used
LA-180 conductor	0.832 units/km	To be included only if used
LA-280 conductor	0.95 units/km	To be included only if used
LA-380 conductor	0.125 units/km	To be included only if used

Figure 11 shows the 5 catenary/autotransformers combinations chosen (marked with arrows), that have been evaluated for sector 30km, 40km and 50km long.

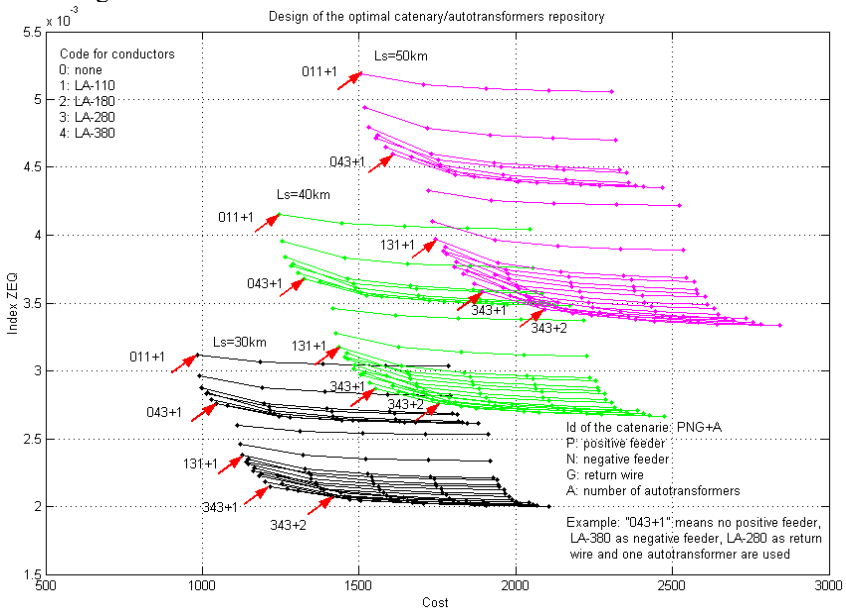


Figure 11: Effect of shortening length of cells.

Each curve represents the effect of adding up to 5 autotransformers to the catenary specified, for the given sector length.

As shown in Figure 11, the efficiency gain of upgrading catenary (change to a lower curve of the same family) is normally a more efficient option than adding an extra autotransformer (go to the following point of the same curve).

It can also be observed that the efficiency of adding autotransformers decreases very quickly and, thus, none of the chosen configurations uses more than 2 autotransformers.

## 6 Conclusions

Using the mono-voltage equivalent model of bi-voltage systems, the existing relationships between the location of autotransformers and the choice of the catenary have been analyzed. Furthermore, a procedure has been proposed to obtain a reduced repository of combinations of catenary types and number of autotransformers in which efficiency and cost are optimized

As a study case, the proposed optimization procedure has been used to obtain a reduced set of catenaries and autotransformers based in the catenary C-350, which has been used in the Madrid – Barcelona – French border new high-speed line. The solution obtained in the study case suggests that upgrading catenary is normally a much more efficient option than adding an extra autotransformer. The reason for that is that the marginal cost of using larger conductors in the catenary can be neglected compared to the marginal cost of adding an autotransformer.

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